



Vague Reservoir Water Quality Evaluation Method Based on Distance

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ABSTRACT

This paper provides the distance-based vague set pattern recognition method for research on water quality evaluation problems using the new formula of the distance between vague sets, and shows with examples that this method not only is simple in calculation, but also provides the results following the practical situation.

INTRODUCTION

At present, many methods can be used to study the reservoir water quality evaluation problems, such as the comprehensive index method (Feng Hui-fang et al. 2010), artificial neural network method (Li Jia-xuan et al. 2010), and grey theory method (Zhang Xin-yu et al. 2011) and so on. In fact, each method has strengths and weaknesses. Water samples do not have strict properties and have uncertainty in terms of form and category, so the reservoir water quality evaluation problems cannot be researched with a perfect method. The vague set theory proposed by Gao & Buehrer (1993) is a promotion of the vague set theory. So far, some methods have been put forward and applied to solve many practical problems, such as the vague set pattern recognition method (Wang Hongxu et al. 2011), vague set TOPSIS method (Yu Fang 2011) and vague set fault diagnosis method (Ye Jun 2006) etc. Of course, these methods also have strengths and weaknesses. This paper will research the water quality evaluation of a reservoir in Hainan Province using the vague reservoir water quality evaluation method based on the distance between vague sets.

CONCEPT OF VAGUE SET

Definition 1: Suppose that the vague set A in the discourse domain X is expressed as the true membership function t_A and false membership function f_A ($t_A: X \rightarrow [0,1], f_A: X \rightarrow [0,1]$). Where, for any $x \in X$, there is $t_A(x) + f_A(x) \leq 1$. $t_A(x)$ is the lower bound of the affirmative membership degree derived by the evidence supporting x , and $f_A(x)$ is the lower bound of the denial membership degree derived by the evidence against x . Membership degree of the element x in the

vague set A is defined by a subinterval $[t_A(x), 1 - f_A(x)]$ of the interval $[0,1]$. This interval is known as the vague value of the element x in A , and is denoted as $A(x) = [t_A(x), 1 - f_A(x)]$ or $x = [t_A(x), 1 - f_A(x)]$, or $x = [t_x, 1 - f_x]$. When is a discrete discourse domain, the vague set A is denoted as:

$$A = \{ [t_A(x_1), 1 - f_A(x_1)], [t_A(x_2), 1 - f_A(x_2)], \\ \mathbf{L}, [t_A(x_n), 1 - f_A(x_n)] \}$$

NEW DISTANCE BETWEEN VAGUE SETS

Definition 2 (Liu Hua-wen et al. 2004): Suppose that $x = [t_x, 1 - f_x]$ is a vague value, a data mining method of it is:

$$\text{Define } t_x^{(0)} = t_x, f_x^{(0)} = f_x, p_x^{(0)} = p_x; \\ t_x^{(m)} = t_x(1 + p_x + p_x^2 + \mathbf{L} + p_x^m), f_x^{(m)} = f_x(1 + p_x + \\ \mathbf{L} + p_x^m), p_x^{(m)} = p_x^{m+1}, m = 1, 2, \mathbf{L}.$$

Theorem 1 (Liu Hua-wen et al. 2004):

$$x_x^{(m)} = [t_x^{(m)}, 1 - f_x^{(m)}] \quad (m = 0, 1, 2, \mathbf{L}.) \text{ is a vague value.}$$

Definition 3 Suppose that there is a vague set in the discourse domain $X = \{x_1, x_2, \mathbf{L}, x_n\}$

$$R = ([t_R(x_1), 1 - f_R(x_1)], [t_R(x_2), 1 - f_R(x_2)], \\ \mathbf{L}, [t_R(x_n), 1 - f_R(x_n)]) \\ G = ([t_G(x_1), 1 - f_G(x_1)], [t_G(x_2), 1 - f_G(x_2)], \\ \mathbf{L}, [t_G(x_n), 1 - f_G(x_n)])$$

respectively simplified as:

$$R = ([t_{x1}, 1 - f_{x1}], [t_{x2}, 1 - f_{x2}], \mathbf{L}, [t_{xn}, 1 - f_{xn}]), \\ G = ([t_{y1}, 1 - f_{y1}], \mathbf{L}, [t_{yn}, 1 - f_{yn}]),$$

Suppose that $x = [t_x, 1 - f_x]$, $y = [t_y, 1 - f_y]$ is a vague value. If the formula $D(R, G)$ satisfies the following rule, then the formula is known as the distance between vague sets R and G :

- (R1). $0 \leq D(R, G) \leq 1$;
- (R2). $D(R, G) = D(G, R)$;
- (R3). If $H = ([t_{z1}, 1 - f_{z1}], \mathbf{L}, [t_{zn}, 1 - f_{zn}])$, then $D(R, H) \leq D(R, G) + D(G, H)$;
- (R4). If $R = \{[0, 1], \mathbf{L}, [0, 1]\}$, $G = \{[1, 1], \mathbf{L}, [1, 1]\}$, or $R = \{[1, 1]\}, \dots, \{[1, 1]\}$, $G = \{[0, 1]\}, \dots, \{[0, 1]\}$ or $R = \{[0, 1], \mathbf{L}, [0, 1]\}$, $G = \{[0, 0], \mathbf{L}, [0, 0]\}$, or $R = \{[0, 0], \mathbf{L}, [0, 0]\}$, $G = \{[0, 1], \mathbf{L}, [0, 1]\}$. then $D(R, G) = 1$;
- (R5) $D(R, G) = 0 \Leftrightarrow R = G, p_{xi} = p_{yi} = 0, (i = 1, 2, \mathbf{L}, n)$.

Theorem 2 Then the following formula $D^{(m)}(R, G)$ is the distance between vague sets R and G :

$$D^{(m)}(R, G) = \frac{1}{n} \sum_{i=1}^n \frac{|S_{xi}^{(m)} - S_{yi}^{(m)}| + |K_{xi}^{(m)} - K_{yi}^{(m)}| + P_{xi}^{(m)} + P_{yi}^{(m)}}{2} \dots(1)$$

VAGUE WATER QUALITY EVALUATION METHOD BASED ON THE DISTANCE BETWEEN VAGUE SETS

The vague set pattern recognition method in literature (Wang Hongxu et al. 2011) is based on similarity measure. Here, it is modified to the distance-based vague set pattern recognition method, and provides the following processes especially for water quality evaluation problems.

Specific application steps of the distance-based vague set pattern recognition method are as follows: Step 1: Establish the index set; $X = \{x_1, x_2, \mathbf{L}, x_n\}$; Step 2: Establish the sample set $G = \{G_1, G_2, \mathbf{L}, G_h\}$ for evaluation in the index set; establish the standard sample set $R = \{R_1, R_2, \mathbf{L}, R_k\}$, provides the original data; Step 3: Enter the vague environment, that is, convert the original data into vague set data; Step 4: Compute the distance between the vague set of the sample $G_j (j = 1, 2, \mathbf{L}, h)$ for evaluation and that of the standard sample $R_i (i = 1, 2, \mathbf{L}, k)$, and rank the distance in ascending order $D(G_j, R_i) \leq D(G_j, R_2) \leq \mathbf{L} \leq D(G_j, R_k) (j = 1, 2, \mathbf{L}, h)$ then the sample G_j for evaluation belongs to the standard sample R_i . Where, $i_1 i_2 \mathbf{L} i_k$ is non-repeated full permutation of $1, 2, \mathbf{L}, k$.

APPLICATION EXAMPLE

The distance-based vague set pattern recognition method is used for recognition and evaluation of a large reservoir in

Hainan with the monitoring data at 2 centres in the monitoring section of the reservoir as the sample for evaluation, and with the national standard as the standard sample. Specific steps are as follows.

Establish the index set. Establish the index set. Here, $X = \{x_1, x_2, x_3, x_4\}$, where, x_1 is the dissolved oxygen: $mg/L \geq$, x_2 is the permanganate index: $mg/L \leq$, x_3 is ammonia nitrogen: $mg/L \leq$, and x_4 is total phosphorus $mg/L \leq$.

Establish the sample set for evaluation and standard sample set. The standard sample set $R = \{R_1, R_2, R_3, R_4, R_5\}$ is established in the index set. Where, water quality R_1 is of the Class I, R_2 is of the Class II, R_3 is of the Class III, R_4 is of the Class IV, and R_5 is of the Class V. The corresponding pollution levels are respectively clean water, no pollution, mild pollution, medium pollution and severe pollution. The surface water environment quality standards in our country are taken from literature (The State Environmental Protection Administration 2002), and their specific data are provided in Table 1.

The sample set $G = \{G_1, G_2\}$ for evaluation is established in the index set. Where, G_1 is the sampling site at the centre of Yacha in the reservoir, and G_2 is the sample point at the center of Nanfeng in the reservoir. The sampling data are provided in Table 1.

Enter vague environment. Literature (Hongxu Wang et al. 2012) provides the formula of converting non-negative individual value data to vague value data, which is in the following form in this example.

$$x_{ij} = [t_{ij}, 1 - f_{ij}] = \left(\frac{x_{ij}}{x_{i\max}}, 1 - \left(\frac{x_{ij}}{x_{i\max}} \right)^{\frac{1}{2}} \right) \dots(2)$$

Where, $x_{i\max} = \max \{x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5}, x_{i6}, x_{i7}\}, i = 1, 2, 3, 4$.

According to Formula (2), the original data in Table 1 are converted to the vague value data in Table 2, which provides the vague value data of the standard samples and of the samples for evaluation. For example:

$$R_1 = \{[0.989, 0.964], [0.133, 0.365], [0.075, 0.274], [0.050, 0.224]\},$$

$$G_1 = \{[1.000, 1.000], [0.182, 0.427], [0.090, 0.300], [0.095, 0.308]\}.$$

Compute the distance between the vague set of samples for evaluation and the vague set of standard samples: According to Formula (1), the distance between the vague set of the sample $G_j (j = 1, 2)$ for evaluation and the vague set of the standard sample $R_i (i = 1, 2, 3, 4, 5)$ is calculated

Table 1: Standard sample and samples for evaluation of the surface water environment quality in China (original data).

	R_1	R_2	R_3	R_4	R_5	G_1	G_2
x_1	7.5	6	5	3	2	8.07	7.25
x_2	2	4	6	10	15	2.73	1.46
x_3	0.15	0.50	1.00	1.50	2.00	0.179	0.152
x_4	0.010	0.025	0.050	0.100	0.200	0.019	0.010

Table 2: Standard sample and samples for evaluation of the surface water environment quality in China (vague value data).

	R_1	R_2	R_3	R_4	R_5	G_1	G_2
x_1	[0.989,0.964] [0.743,0.862] [0.620,0.787] [0.372,0.610] [0.267,0.571] [1.000,1.000] [0.989,0.948]						
x_2	[[0.133,0.365] [0.267,0.517] [0.400,0.632] [0.667,0.817] [1.000,1.000] [0.182,0.427] [0.097,0.311]						
x_3	[0.075,0.274] [0.250,0.500] [0.500,0.707] [0.750,0.866] [1.000,1.000] [0.090,0.300] [0.076,0.276]						
x_4	[0.050,0.224] [0.125,0.354] [0.250,0.500] [0.500,0.707] [1.000,1.000] [0.095,0.308] [0.050,0.224]						

when the parameter $m = 2$. The results are as follows:

$$D^{(2)}(R_1, G_1) = 0.036, D^{(2)}(R_2, G_1) = 0.110, D^{(2)}(R_3, G_1) = 0.217, \dots(3)$$

$$D^{(2)}(R_4, G_1) = 0.389, D^{(2)}(R_5, G_1) = 0.533.$$

$$D^{(2)}(R_1, G_2) = 0.016, D^{(2)}(R_2, G_2) = 0.120, D^{(2)}(R_3, G_2) = 0.240, \dots(4)$$

$$D^{(2)}(R_4, G_2) = 0.418, D^{(2)}(R_5, G_2) = 0.556.$$

The following formula can be obtained from the Formula (3)

$$D^{(2)}(R_1, G_1) < D^{(2)}(R_2, G_1) < D^{(2)}(R_3, G_1) < D^{(2)}(R_4, G_1) < D^{(2)}(R_5, G_1).$$

Hence, G_1 belongs to the standard sample R_1 and clean water quality of Class I. The following formula can be obtained from the Formula (4)

$$D^{(2)}(R_1, G_2) < D^{(2)}(R_2, G_2) < D^{(2)}(R_3, G_2) < D^{(2)}(R_4, G_2) < D^{(2)}(R_5, G_2).$$

Hence, G_2 belongs to the standard sample R_1 and clean water quality of Class I.

CONCLUSIONS

According to the distance-based vague set pattern recognition

method, data of the sample for evaluation of 2 sampling sites at the center of a reservoir in Hainan Province all show clean water quality of Class I, and are exactly the same as the evaluation of the professional personnel on the reservoir.

The distance-based vague set pattern recognition method provided in this paper is used to study the water quality evaluation problems, and provides a new way for study of similar problems. The method is the vague set pattern recognition method based on the new distance of vague set (1), and not only is simple in calculation, but also provides the results following the practical situation.

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